Bounds on the variance of randomly shifted lattice rules

Vasile Sinescu

Katholieke Universiteit Leuven

Joint work with: Pierre L'Ecuyer

The variance of randomly shifted lattice rules for numerical multiple integration can be expressed by the Fourier coefficients over the dual lattice. Bounds on the variance can then be obtained by making certain assumptions on the smoothness of the integrands, which is reflected in the rate of decay of their corresponding Fourier coefficients.

Here we only assume that the integrands are square integrable. We allow our integrands to have the associated Fourier series not absolutely convergent. Thus, our assumptions are weaker than the usual assumptions made on the space of functions that are integrated by lattice rules. We obtain a bound for the variance that is the same as the worst-case error in weighted Korobov spaces, but can be used for a larger class of integrands. It is known that the square worst-case error in Korobov spaces converges as $O(n^{-\alpha}(\log n)^{d\alpha})$ for $\alpha > 1$. We show that we can obtain a refined convergence order of $O(n^{-\beta}(\log\log n)^{\beta})$, where $1 \leq \beta < \alpha$, which has the merit of being independent of the dimension.

We then examine the impact of these randomly shifted lattice rules for some problems that arise in practice and lead to integrals that fit our assumptions. Such examples are the likelihood estimation in a mixed logit model and the payoff function of a barrier option in finance.

Under the assumption that weights are general and the number of points is composite, we construct good lattice rules by using the well known 'component by component' technique or some simpler randomised algorithms and compare the performances of these algorithms.